Phenomenological Applications of k_T factorization

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Abstract

We discuss applications of the perturbative QCD approach in the exclusive non-leptonic two body B-meson decays. We briefly review its ingredients and some important theoretical issues on the factorization approach. PQCD results are compatible with present experimantal data for charmless B-meson decays. We predict the possibility of large direct CP asymmetry in $B^0 \to \pi^+\pi^-$ ($23 \pm 7\%$) and $B^0 \to K^+\pi^-$ ($-17 \pm 5\%$). We also investigate the Branching ratios, CP asymmetry and isopsin symmetry breaking in radiative $B \to (K^*/\rho)\gamma$ decays.

PACS numbers:

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I. INTRODUCTION

The aim of the study on weak decay in B-meson is two folds: (1) To determine precisely the elements of Cabibbo-Kobayashi-Maskawa (CKM) matrix[1, 2] and to explore the origin of CP-violation at a low energy scale, (2) To understand strong interaction physics related to the confinement of quarks and gluons within hadrons.

The two tasks complement each other. An understanding of the connection between quarks and hadron properties is a necessary prerequeste for a precise determination of CKM matrix elements and CP-violating phases, so called Kobayashi-Maskawa(KM) phase[2].

The theoretical description of hadronic weak decays is difficult since nonperturbative QCD interactions is involved. This makes a difficult to interpret correctly data from asymmetric B-factories and to seek the origin of CP violation. In the case of B-meson decays into two light mesons, we can explain roughly branching ratios by using the factorization approximation [3, 4]. Since B-meson is quite heavy, when it decays into two light mesons, the final-state mesons are moving so fast that it is difficult to exchange gluons between final-state mesons. So we can express the amplitude in terms of the product of weak decay constant and transition form factors by the factorization (color-transparancy) argument[5, 6]. In this approach we neglect non-factorizable contributions and a power suppressed annihilation contributions. Because of this weakness, asymmetry of CP violation can not be predicted correctly.

Recently two different QCD approaches beyond naive and general factorization assumption [3, 4, 9, 10] was proposed: (1) QCD-factorization in the heavy quark limit [11, 12] in which non-factorizable terms and a_i are calculable in some cases. (2) A Novel PQCD approach [13, 15, 16] including the resummation effects of the transverse momentum carried by partons inside meson. In this review paper, we discuss some important theoretical issues in the PQCD factorization and numerical results for charmless B-decays at the section 3-7. In section 8 we present the PQCD results of the radiative B-decays $B \to K^*\gamma, \rho\gamma$.

II. k_T FACTORIZATION VS COLLINEAR FACTORIZATION

Let's start to review shortly on the developement of theoretical methods of exclusive hadronic B-meson two-body decays, and make a comparision between different frameworks

of factorization.

The theoretical basis how we can explain nonleptonic B-meson decays is origined from the color-transparancy argument[5, 6]: *Since b-quark decays into light quarks energetically (> 1 GeV), the produced quark-antiquark pair doesn't have enough time to evolve to the real size hadronic entity, but remains a small size bound state with a correspondingly small chromomagnetic moment which suppress in QCD interaction between final state mesons.*

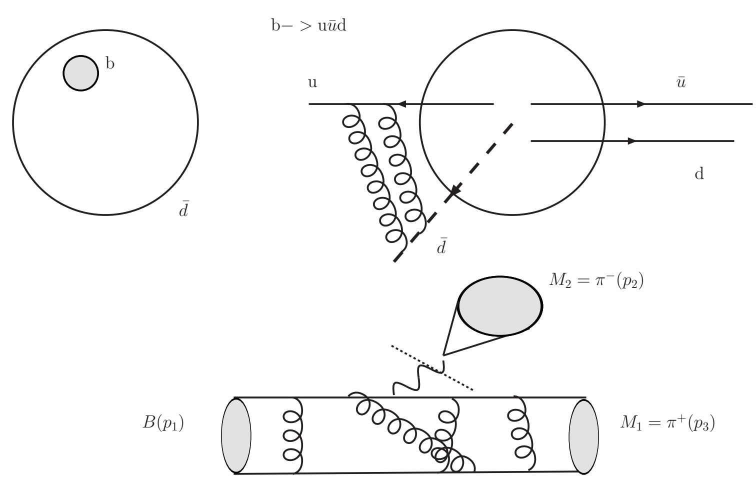

FIG. 1: Color transparancy arguement of the nonleptonic B-decays.

As shown in Fig. 1, when we consider B-meson decays into two final state pions, the matrix elements of 0_1 can be expressed by a simple way as so called the naive factorization method[3, 4]:

$$\begin{aligned} <\pi(p_2)\pi(p_3)|0_1(\mu)|B(p_1)> &\sim <\pi(p_2)|(d_i q_i)_{V-A}|0><\pi(p_3)|(q_j b_j)_{V-A}|B(p_1)> \\ &= \qquad f_\pi \qquad \otimes \qquad F^{B\pi}(q^2 = M_\pi^2) \end{aligned} \tag{1}$$

In this way, only factorizable part was considered, but not nonfactorizable part. In 1996, the generalied factorization approach was developped by A. Kamal[7] and H.-Y. Cheng[8], which included non-factorizable contributions into the effective wilson coefficients by assuming $NF = \chi \otimes F$. The generalized factorization has a weak point to predict the CP asymmetry, since they considered non-factorizable part to be real, however it is complex in general. After then, QCD-factorization was proposed by Beneke et al.[11], which is an improved form of

naive factorization approach. When we consider $B \to M_1 M_2$ with recoiled M_1 and emitted M_2(light or quarkonium), soft gluon exchanged effects are confined to (BM_1) system and only hard interactions between (BM_1) and M_2 survive in $m_b \to \infty$ limit which is calculable perturbatively. The decay amplitude can be written as:

$$\text{Decay Amp} = \text{Amp}_{(naive\ fact)} \otimes [1 + O(\alpha_s) + O(\frac{\Lambda_{QCD}}{m_b})]. \tag{2}$$

In principle, nonfactorizable part and a_i are calculable in the heavy quark limit within the leading twist, however it is diverged at the end point with the twist-3 contributions and even the leading twist contribution in the annihilation diagram. To solve this end-point sigularity problem, PQCD approach was proposed by Keum et al.[13], in which hard gluon exchanged contributions is dominant even in the (BM_1) transition form factor. We will discuss the detail of PQCD approach in the next section.

Now we explain how to derive collinear and k_T factorization theorems for the pion form factor involved in the scattering process $\pi(P_1)\gamma^*(q) \to \pi(P_2)$. The momenta are chosen in the light-cone coordinates as $P_1 = (P_1^+, 0, \mathbf{0}_T)$, $P_2 = (0, P_2^-, \mathbf{0}_T)$, and $Q^2 = -q^2$. At leading order, $O(\alpha_s)$, shown in Fig. 2(a), the hard kernel is proportional to $H^{(0)}(x_1, x_2) \propto -1/(x_1 P_1 - x_2 P_2)^2 = 1/(x_1 x_2 Q^2)$. Here x_1 and x_2 are the parton momentum fractions carried by the lower quarks in the incoming and outgoing pions, respectively. At next-to-leading order, $O(\alpha_s^2)$, collinear divergences are generated in loop integrals, and need to be factorized into the pion wave function. In the collinear region with the loop momentum l parallel to P_1, we have an on-shell gluon $l^2 \sim P_1^2 \sim O(\Lambda^2)$ with the hierachy of the components, $l^+ \sim P_1^+ \gg l_T \sim \Lambda \gg l^- \sim \Lambda^2/P_1^+$.

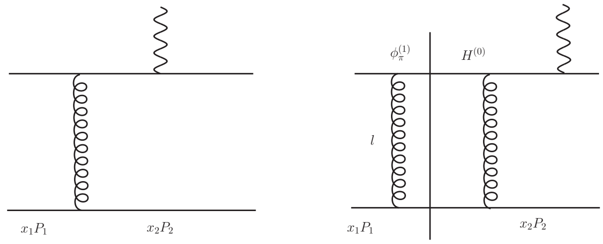

FIG. 2: (a) Lowest-order diagram for F_π. (b) Radiative correction to (a).

An example of next-to-leading-order diagrams is shown in Fig. 2(b). The factorization of Fig. 2(b) is trivial: one performs the Fierz transformation to separate the fermion flows, so that the right-hand side of the cut corresponds to the lowest-order hard kernel $H^{(0)}$. Since the loop momentum l flows into the hard gluon, we have the gluon momentum $x_1P_1-x_2P_2+l$ and

$$H^{(0)} \propto \frac{-1}{(x_1P_1-x_2P_2)^2+2x_1P_1^+l^- -2x_2P_2^-l^+ +2l^+l^- -l_T^2} \ . \tag{3}$$

Dropping l^- and l_T as a collinear approximation, the above expression reduces to

$$H^{(0)}(\xi_1,x_2) \propto \frac{1}{2x_1x_2P_1^+P_2^- +2x_2P_2^-l^+} \equiv \frac{1}{\xi_1 x_2 Q^2} \ , \tag{4}$$

where $\xi_1 = x_1 + l^+/P_1^+$ is the parton momentum fraction modified by the collinear gluon exchange. The left-hand side of the cut then contributes to the $O(\alpha_s)$ distribution amplitude $\phi_\pi^{(1)}(\xi_1)$, which contains the integration over l^- and l_T. Therefore, factorization to all orders gives a convolution only in the longitudinal components of parton momentum,

$$F_\pi = \int d\xi_1 d\xi_2 \phi_\pi(\xi_1) H(\xi_1,\xi_2)\phi_\pi(\xi_2) \ . \tag{5}$$

In the region with small parton momentum fractions, the hard scale $x_1x_2Q^2$ is not large. In this case one may drop only l^-, and keep l_T in $H^{(0)}$. This weaker approximation gives [14]

$$H^{(0)}(\xi_1,x_2,l_T) \propto \frac{1}{2(x_1+l^+/P_1^+)x_2P_1^+P_2^- + l_T^2} \equiv \frac{1}{\xi_1 x_2 Q^2 + l_T^2} \ , \tag{6}$$

which acquires a dependence on a transverse momentum. We factorize the left-hand side of the cut in Fig. 2(b) into the $O(\alpha_s)$ wave function $\phi_\pi^{(1)}(\xi_1,l_T)$, which involves the integration over l^-. It is understood that the collinear gluon exchange not only modifies the momentum fraction, but introduces the transverse momentum dependence of the pion wave function. Extending the above procedure to all orders, we derive the k_T factorization,

$$F_\pi = \int d\xi_1 d\xi_2 d^2k_{1T} d^2k_{2T} \phi_\pi(\xi_1,k_{1T}) H(\xi_1,\xi_2,k_{1T},k_{2T}) \phi_\pi(\xi_2,k_{2T}) \ . \tag{7}$$

III. INGREDIENTS OF k_T FACTORIZATION APPROACH

Factorization in PQCD: The idea of pertubative QCD is as follows: When heavy B-meson decays into two light mesons, the hard process is dominant. Since two light mesons fly

so fast with large momentum, it is reasonable assumptions that the final-state interaction is not important for charmless B-decays. Hard gluons are needed to boost the resting spectator quark to get large momentum and finally to hadronize a fast moving final meson. So the dominant process is that one hard gluon is exchanged between specator quark and other four quarks.

Let's start with the lowest-order diagram of $B \to K\pi$. The soft divergences in the $B \to \pi$ form factor can be factorized into a light-cone B meson wave function, and the collinear divergences can be absorbed into a pion distribution amplitude. The finite pieces of them is absorbed into the hard part. Then in the natural way we can factorize amplitude into two pieces: $G \equiv H(Q,\mu) \otimes \Phi(m,\mu)$ where H stands for hard part which is calculable with a perturbative way. Φ represents a product of wave functions which contains all the nonperturbative dynamics.

PQCD adopt the three scale factorization theorem [17] based on the perturbative QCD formalism by Brodsky and Lepage [18], and Botts and Sterman [19], with the inclusion of the transverse momentum components carried by partons inside meson.

We have three different scales: electroweak scale: M_W, hard interaction scale: $t \sim O(\sqrt{(}\bar{\Lambda} m_b))$, and the factorization scale: $1/b$ where b is the conjugate variable of parton transverse momenta. The dynamics below $1/b$ is completely non-perturbative and can be parameterized into meson wave funtions which are universal and process independent. In our analysis we use the results of light-cone distribution amplitudes (LCDAs) by Ball [20, 21] with the light-cone sum rule calculation.

The ampltitude in PQCD is expressed as

$$
\begin{aligned}
\langle M_1 M_2 | C_k(t) \mathcal{O}_k | B \rangle \;=\; & \int [dx] \int \left[\frac{d^2\vec{b}}{4\pi} \right] \Phi^*_{M_1}(x_2, \vec{b}_2)\, \Phi^*_{M_2}(x_3, \vec{b}_3)\, C_k(t) \\
& \otimes\; H_k(\{x\}, \{\vec{b}\}, M_B) \Phi_B(x_1, \vec{b}_1)\; S_t(\{x\})\; e^{-S\left(\{x\},\{\vec{b}\},M_B\right)} \qquad (8)
\end{aligned}
$$

with the sudakov suppressed factor: $S = S_B(x_1 P_1^+, b_1) + S_{M_1}(x_2 P_2^-, b_2) + S_{M_1}((1 - x_2)P_2^-, b_2) + ...$ and the threshold resummation factor $S_t(x)$. Here $C(t)$ are Wilson coefficients, $\Phi(x)$ are meson LCDAs and variable t is the factorized scale in hard part.

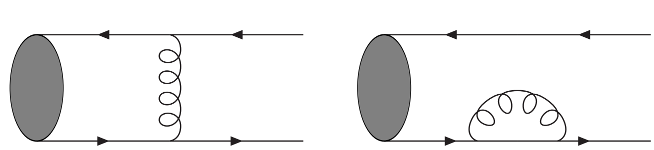

FIG. 3: The diagrams generate double logarithm corrections for the sudakov resummation.

Sudakov Suppression Effects: When we include $k_\perp$, the double logarithms $\ln^2(Pb)$ are generated from the overlap of collinear and soft divergence in radiative corrections to meson wave functions(See figure 3), where P is the dominant light-cone component of a meson momentum. The resummation of these double logarithms leads to a Sudakov form factor $exp[-s(P,b)]$ in Eq.(8), which suppresses the long distance contributions in the large b region, and vanishes as $b > 1/\Lambda_{QCD}$. This suppression renders $k_\perp^2$ flowing into the hard

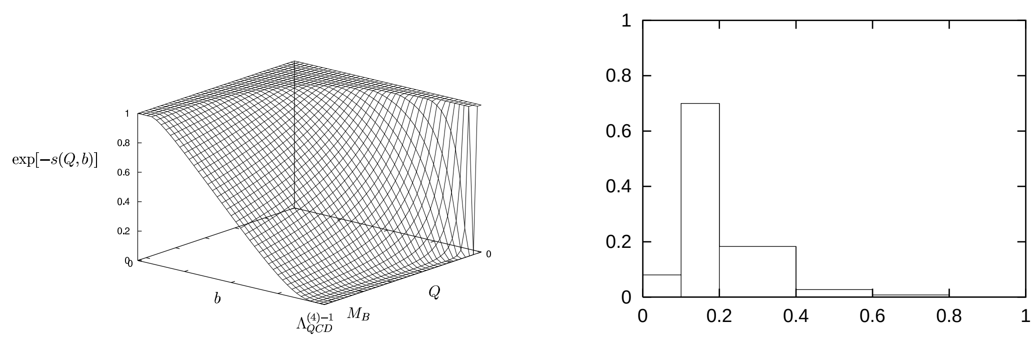

FIG. 4: (a)Sudakov suppression factor (b)Fractional contribution to the $B \to \pi$ transition form factor $F^{B\pi}$ as a function of $\alpha_s(t)/\pi$.

amplitudes of order

$$< k_\perp^2 > \sim O(\bar{\Lambda} M_B) \; . \tag{9}$$

The off-shellness of internal particles then remain of $O(\bar{\Lambda} M_B)$ even in the end-point region, and the singularities are removed. This mechanism is so-called Sudakov suppression(See figure 4-a).

Du *et al.* have studied the Sudakov effects in the evaluation of nonfactorizable amplitudes [22]. If equating these amplitudes with Sudakov suppression included to the parametrization in QCDF, it was observed that the corresponding cutoffs are located in the reasonable range proposed by Beneke *et al.* [12]. Sachrajda *et al.* have expressed an opposite opinion on the effect of Sudakov suppression in [23]. However, their conclusion was drawn based on a very sharp B meson wave function, which is not favored by experimental data.

Here I would like to comment on the negative opinions on the large $k_\perp^2 \sim O(\bar{\Lambda} M_B)$. It is easy to understand the increase of $k_\perp^2$ from $O(\bar{\Lambda}^2)$, carried by the valence quarks which

just come out of the initial meson wave functions, to $O(\bar{\Lambda} M_B)$, carried by the quarks which are involved in the hard weak decays. Consider the simple deeply inelastic scattering of a hadron. The transverse momentum $k_\perp$ carried by a parton, which just come out of the hadron distribution function, is initially small. After infinite many gluon radiations, $k_\perp$ becomes of $O(Q)$, when the parton is scattered by the highly virtual photon, where Q is the large momentum transfer from the photon. The evolution of the hadron distribution function from the low scale to Q is described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [24, 25]. The mechanism of the DGLAP evolution in DIS is similar to that of the Sudakov evolution in exclusive B meson decays. The difference is only that the former is the consequence of the single-logarithm resummation, while the latter is the consequence of the double-logarithm resummation.

By including Sudakov effects, all contributions of the $B \to \pi$ form factor comes from the region with $\alpha_s/\pi < 0.3$ [15] as shown in Figure 4(b). It indicate that our PQCD results are well within the perturbative region.

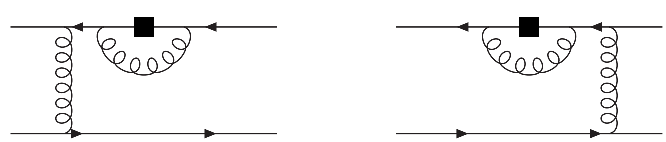

FIG. 5: The diagrams generate double logarithm corrections for the threshold resummation.

Threshold Resummation: The other double logarithm is $\alpha_s \ln^2(1/x)$ from the end point region of the momentum fraction x [26]. This double logarithm is generated by the corrections of the hard part in Figure 5. This double logarithm can be factored out of the hard amplitude systematically, and its resummation introduces a Sudakov factor $S_t(x) = 1.78[x(1-x)]^c$ with $c = 0.3$ into PQCD factorization formula. The Sudakov factor from threshold resummation is universal, independent of flavors of internal quarks, the twists and topologies of hard amplitudes, and the decay modes.

Threshold resummation[26] and $k_\perp$ resummation [19, 27, 28] arise from different subprocesses in PQCD factorization and suppresses the end-point contributions, making PQCD evaluation of exclusive B meson decays reliable. Without these resummation effects, the PQCD predictions for the $B \to K$ form factors are infrared divergent. If including only

$k_\perp$ resummation, the PQCD predictions are finite. However, the two-parton twist-3 contributions are still huge, so that the $B \to K$ form factors have an unreasonably large value $F^{BK} \sim 0.57$ at maximal recoil. The reason is that the double logarithms $\alpha_s \ln^2 x$ have not been organized. If including both resummations, we obtain the reasonable result $F^{BK} \sim 0.35$ as shown in Figure 6.. These studies indicate the importance of resummations in PQCD analyses of B meson decays. In conclusion, if the PQCD analysis of the heavy-to-light form factors is performed self-consistently, there exist no end-point singularities, and both twist-2 and twist-3 contributions are well-behaved.

Amplitudes	twist-2 contribution	Twist-3 contribution	Total
$Re(f_\pi F^T)$	$3.44 \cdot 10^{-2}$	$5.00 \cdot 10^{-2}$	$8.44 \cdot 10^{-2}$
$Im(f_\pi F^T)$	–	–	–
$Re(f_\pi F^P)$	$-1.26 \cdot 10^{-3}$	$-4.76 \cdot 10^{-3}$	$-6.02 \cdot 10^{-3}$
$Im(f_\pi F^P)$	–	–	–
$Re(f_B F_a^P)$	$2.52 \cdot 10^{-6}$	$-3.30 \cdot 10^{-4}$	$-3.33 \cdot 10^{-4}$
$Im(f_B F_a^P)$	$8.72 \cdot 10^{-7}$	$3.81 \cdot 10^{-3}$	$3.81 \cdot 10^{-3}$
$Re(M^T)$	$7.26 \cdot 10^{-4}$	$-1.39 \cdot 10^{-6}$	$-7.25 \cdot 10^{-4}$
$Im(M^T)$	$-1.62 \cdot 10^{-3}$	$-2.91 \cdot 10^{-4}$	$1.33 \cdot 10^{-3}$
$Re(M^P)$	$-1.67 \cdot 10^{-5}$	$-1.47 \cdot 10^{-7}$	$1.66 \cdot 10^{-5}$
$Im(M^P)$	$-3.52 \cdot 10^{-5}$	$6.56 \cdot 10^{-6}$	$-2.87 \cdot 10^{-5}$
$Re(M_a^P)$	$-7.37 \cdot 10^{-5}$	$2.50 \cdot 10^{-6}$	$-7.12 \cdot 10^{-5}$
$Im(M_a^P)$	$-3.13 \cdot 10^{-5}$	$-2.04 \cdot 10^{-5}$	$-5.17 \cdot 10^{-5}$

TABLE I: Amplitudes for the $B_d^0 \to \pi^+\pi^-$ decay where F (M) denotes factorizable (nonfactorizable) contributions, P (T) denotes the penguin (tree) contributions, and a denotes the annihilation contributions. Here we adopted $\phi_3 = 80^0$, $R_b = 0.38$, $m_0^\pi = 1.4\,GeV$ and $\omega_B = 0.40\,GeV$.

Power Counting Rule in PQCD: The power behaviors of various topologies of diagrams for two-body nonleptonic B meson decays with the Sudakov effects taken into account has been discussed in details in [29]. The relative importance is summarized below:

$$\text{emission} : \text{annihilation} : \text{nonfactorizable} = 1 : \frac{2m_0}{M_B} : \frac{\bar{\Lambda}}{M_B} , \tag{10}$$

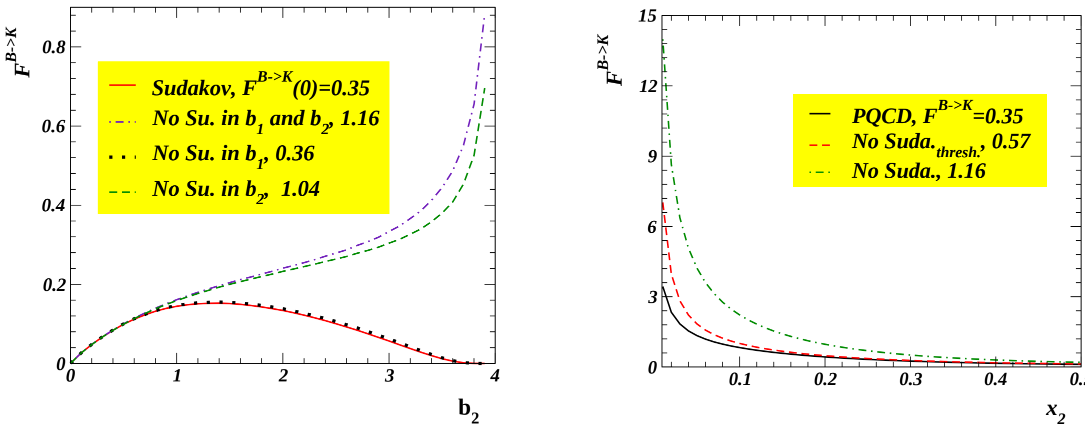

FIG. 6: Sudakov suppression and threshold resummation effects in $B \to K$ transition form factor

with m_0 being the chiral symmetry breaking scale. The scale m_0 appears because the annihilation contributions are dominated by those from the $(V-A)(V+A)$ penguin operators, which survive under helicity suppression. In the heavy quark limit the annihilation and nonfactorizable amplitudes are indeed power-suppressed compared to the factorizable emission ones. Therefore, the PQCD formalism for two-body charmless nonleptonic B meson decays coincides with the factorization approach as $M_B \to \infty$. However, for the physical value $M_B \sim 5$ GeV, the annihilation contributions are essential. In Table 1 and 2 we can easily check the relative size of the different topology in Eq.(10) by the peguin contribution for W-emission $(f_\pi F^P)$, annihilation$(f_B F_a^P)$ and non-factorizable(M^P) contributions as shown in Figure 7. Specially we show the relative size of the different twisted light-cone-distribution-amplitudes (LCDAs) for each topology. We have more sizable twist-3 contributions in the factorizable diagram.

Note that all the above topologies are of the same order in α_s in PQCD. The nonfactorizable amplitudes are down by a power of $1/m_b$, because of the cancellation between a pair of nonfactorizable diagrams, though each of them is of the same power as the factorizable one. I emphasize that it is more appropriate to include the nonfactorizable contributions in a complete formalism. The factorizable internal-W emisson contributions are strongly suppressed by the vanishing Wilson coefficient a_2 in the $B \to J/\psi K^{(*)}$ decays [30], so that nonfactorizable contributions become dominant[31]. In the $B \to D\pi$ decays, there is no soft cancellation between a pair of nonfactorizable diagrams, and nonfactorizable contributions are significant [30].

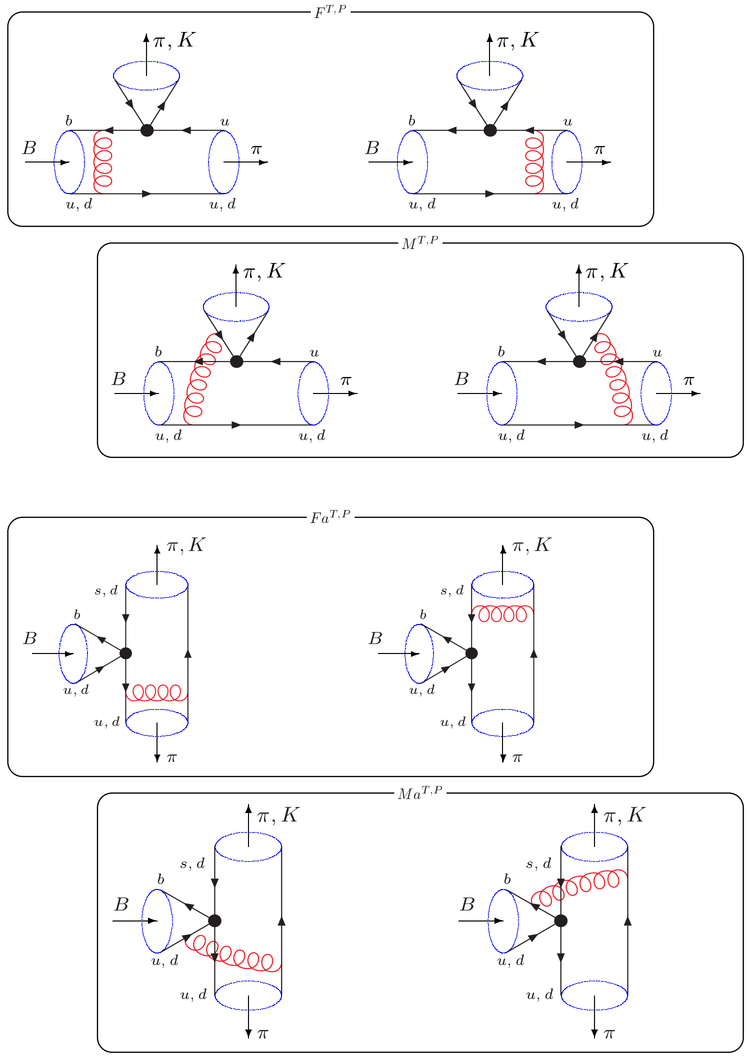

FIG. 7: Feynman diagrams for $B \to \pi\pi$ and $K\pi$.

In QCDF the factorizable and nonfactorizable amplitudes are of the same power in $1/m_b$, but the latter is of next-to-leading order in α_s compared to the former. Hence, QCDF approaches FA in the heavy quark limit in the sense of $\alpha_s \to 0$. Briefly speaking, QCDF and PQCD have different counting rules both in α_s and in $1/m_b$. The former approaches

FA logarithmically ($\alpha_s \propto 1/\ln m_b \to 0$), while the latter does linearly ($1/m_b \to 0$).

Amplitudes	Left-handed gluon exchange	Right-handed gluon exchange	Total
$Re(f_K F^T)$	$7.07 \cdot 10^{-2}$	$3.16 \cdot 10^{-2}$	$1.02 \cdot 10^{-1}$
$Im(f_K F^T)$	–	–	–
$Re(f_K F^P)$	$-5.52 \cdot 10^{-3}$	$-2.44 \cdot 10^{-3}$	$-7.96 \cdot 10^{-3}$
$Im(f_K F^P)$	–	–	–
$Re(f_B F_a^P)$	$4.13 \cdot 10^{-4}$	$-6.51 \cdot 10^{-4}$	$-2.38 \cdot 10^{-4}$
$Im(f_B F_a^P)$	$2.73 \cdot 10^{-3}$	$1.68 \cdot 10^{-3}$	$4.41 \cdot 10^{-3}$
$Re(M^T)$	$7.06 \cdot 10^{-3}$	$-7.17 \cdot 10^{-3}$	$-1.11 \cdot 10^{-4}$
$Im(M^T)$	$-1.10 \cdot 10^{-2}$	$1.35 \cdot 10^{-2}$	$2.59 \cdot 10^{-3}$
$Re(M^P)$	$-3.05 \cdot 10^{-4}$	$3.07 \cdot 10^{-4}$	$2.17 \cdot 10^{-6}$
$Im(M^P)$	$4.50 \cdot 10^{-4}$	$-5.29 \cdot 10^{-4}$	$-7.92 \cdot 10^{-5}$
$Re(M_a^P)$	$2.03 \cdot 10^{-5}$	$-1.37 \cdot 10^{-4}$	$-1.16 \cdot 10^{-4}$
$Im(M_a^P)$	$-1.45 \cdot 10^{-5}$	$-1.27 \cdot 10^{-4}$	$-1.42 \cdot 10^{-4}$

TABLE II: Amplitudes for the $B_d^0 \to K^+\pi^-$ decay where F (M) denotes factorizable (nonfactorizable) contributions, P (T) denotes the penguin (tree) contributions, and a denotes the annihilation contributions. Here we adopted $\phi_3 = 80^0$, $R_b = 0.38$.

IV. IMPORTANT THEORETICAL ISSUES

End Point Singularity and Form Factors: If calculating the $B \to \pi$ form factor $F^{B\pi}$ at large recoil using the Brodsky-Lepage formalism [18, 32], a difficulty immediately occurs. The lowest-order diagram for the hard amplitude is proportional to $1/(x_1 x_3^2)$, x_1 being the momentum fraction associated with the spectator quark on the B meson side. If the pion distribution amplitude vanishes like x_3 as $x_3 \to 0$ (in the leading-twist, *i.e.*, twist-2 case), $F^{B\pi}$ is logarithmically divergent. If the pion distribution amplitude is a constant as $x_3 \to 0$ (in the next-to-leading-twist, *i.e.*, twist-3 case), $F^{B\pi}$ even becomes linearly divergent. These end-point singularities have also appeared in the evaluation of the nonfactorizable and annihilation amplitudes in QCDF mentioned above.

When we include small parton transverse momenta $k_\perp$, we have

$$\frac{1}{x_1\ x_3^2 M_B^4} \qquad \rightarrow \qquad \frac{1}{(x_3\, M_B^2 + k_{3\perp}^2)\ [x_1 x_3\, M_B^2 + (k_{1\perp} - k_{3\perp})^2]} \tag{11}$$

and the end-point singularity is smeared out.

In PQCD, we can calculate analytically space-like form factors for $B \rightarrow P, V$ transition and also time-like form factors for the annihilation process [29, 33].

Strong Phases: While stong phases in FA and QCDF come from the Bander-Silverman-Soni (BSS) mechanism[34] and from the final state interaction (FSI), the dominant strong phase in PQCD come from the factorizable annihilation diagram[13, 15, 16] (See Figure 8). In fact, the two sources of strong phases in the FA and QCDF approaches are strongly suppressed by the charm mass threshold and by the end-point behavior of meson wave functions. So the strong phase in QCDF is almost zero without soft-annihilation contributions.

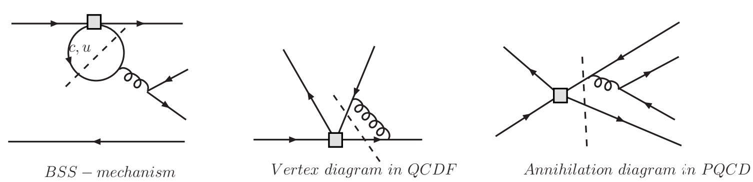

FIG. 8: Different sourses of strong phase: (a)BSS mechanism, (b) Final State Interaction, and (c) Factorizable annihilation.

Dynamical Penguin Enhancement vs Chiral Enhancement: As explained before, the hard scale is about 1.5 GeV. Since the RG evolution of the Wilson coefficients $C_{4,6}(t)$ increase drastically as $t < M_B/2$, while that of $C_{1,2}(t)$ remain almost constant as shown in Figure 9, we can get a large enhancement effects from both wilson coefficents and matrix elements in PQCD.

In general the amplitude can be expressed as

$$Amp \sim [a_{1,2}\ \pm\ a_4\ \pm\ m_0^{P,V}(\mu) a_6] \cdot\ < K\pi|O|B > \tag{12}$$

with the chiral factors $m_0^P(\mu) = m_P^2/[m_1(\mu) + m_2(\mu)]$ for pseudoscalr meson and $m_0^V = m_V$ for vector mesons. To accommodate the $B \rightarrow K\pi$ data in the factorization and QCD-factorization approaches, one relies on the chiral enhancement by increasing the mass m_0 to

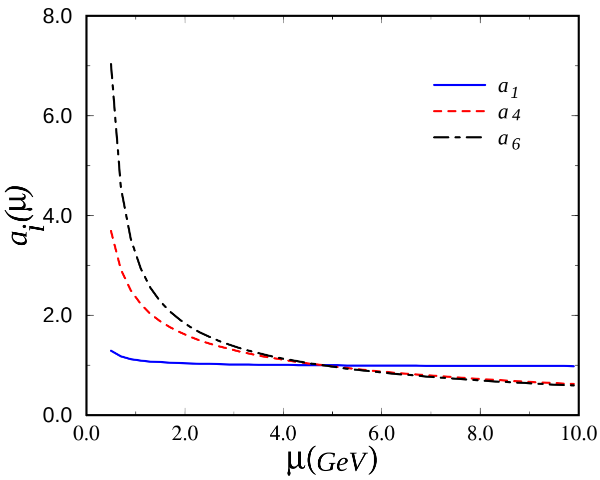

FIG. 9: Dynamical enhancement of Wilson coefficents a_i (i=1,4,6).

as large values about 3 GeV at $\mu = m_b$ scale. So two methods accomodate large branching ratios of $B \to K\pi$ and it is difficult for us to distinguish two different methods in $B \to PP$ decays. However we can do it in $B \to PV$ because there is no chiral factor in LCDAs of the vector meson. We can test whether dynamical enhancement or chiral enhancement is responsible for the large $B \to K\pi$ branching ratios by measuring the $B \to \phi K$ modes. In these modes penguin contributions dominate, such that their branching ratios are insensitive to the variation of the unitarity angle ϕ_3. According to recent works by Cheng *at al.* [35], the branching ratio of $B \to \phi K$ is $(2-7) \times 10^{-6}$ including 30% annihilation contributions in the QCD-factorization approach (QCDF). However PQCD predicts 10×10^{-6} [29, 42]. For $B \to \phi K^*$ decays, QCDF gets about 9×10^{-6}[36], but PQCD have 15×10^{-6}[43]. Because of these small branching ratios for $B \to PV$ and VV decays in the QCD-factorization approach, they can not globally fit the experimental data for $B \to PP, VP$ and VV modes simultaneously with same sets of free parameters (ρ_H, ϕ_H) and (ρ_A, ϕ_A) [37].

Fat Imaginary Penguin in Annihilation: There is a falklore that the annihilation contribution is negligible compared to W-emission one. In this reason the annihilation contribution was not included in the general factorization approach and the first paper on QCD-factorization by Beneke et al. [11]. In fact there is a suppression effect for the operators with structure $(V-A)(V-A)$ because of a mechanism similar to the helicity

suppression for $\pi \to \mu\nu_\mu$. However annihilation from the operators $O_{5,6,7,8}$ with the structure $(S-P)(S+P)$ via Fiertz transformation survive under the helicity suppression and can get large imaginary value. The real part of factorized annihilation contribution becomes small because there is a cancellation between left-handed gluon exchanged one and right-handed gluon exchanged one as shown in Table 2. This mostly pure imaginary value of annihilation is a main source of large CP asymmetry in $B \to \pi^+\pi^-$ and $K^+\pi^-$. In Table 6 we summarize the CP asymmetry in $B \to K(\pi)\pi$ decays.

V. NUMERICAL RESULTS

Branching ratios in Charmless B-decays: The PQCD approach allows us to calculate the amplitudes for charmless B-meson decays in terms of ligh-cone distribution amplitudes upto twist-3. We focus on decays whose branching ratios have already been measured. We take allowed ranges of shape parameter for the B-meson wave funtion as $\omega_B = 0.36 - 0.44$ which accomodate to reasonable form factors, $F^{B\pi}(0) = 0.27 - 0.33$ and $F^{BK}(0) = 0.31 - 0.40$. We use values of chiral factor with $m_0^\pi = 1.3GeV$ and $m_0^K = 1.7GeV$. Finally we obtain branching ratios for $B \to K(\pi)\pi$ [38, 39], $K\phi$ [29, 42] $K^*\phi$[43] and $K^*\pi$[40], which is well agreed with present experimental data in Table 3-5.

CP Asymmetry of $B \to \pi\pi, K\pi$: Because we have a large imaginary contribution from factorized annihilation diagrams in PQCD approach, we predict large CP asymmetry ($\sim 25\%$) in $B^0 \to \pi^+\pi^-$ decays and about -15% CP violation effects in $B^0 \to K^+\pi^-$. The detail prediction is given in Table 6. The CP asymmetry is defined as followings:

$$\begin{aligned} A_{CP}(\triangle t) &= \frac{N(\bar{B} \to \bar{f}) - N(B \to f)}{N(\bar{B} \to \bar{f}) + N(B \to f)} \\ &= S_f\ sin(\triangle m_d \triangle t) - C_f\ Cos(\triangle m_d \triangle t). \end{aligned} \tag{13}$$

Here we notice that the relation between two different definitions: $A_f(Belle) = -C_f(BaBar)$. In our analysis we used the Belle notation. The precise measurement of direct CP asymmetry (both magnitude and sign) is a crucial way to test factorization models which have different sources of strong phases. Our predictions for CP-asymmetry on $B \to K(\pi)\pi$ have a totally opposite sign to those of QCD factorization. Recently it was confirmed as the first evidence of the direct CP-violation in B-decays that the DCP asymmetry in $B \to K^\pm\pi^\mp$ decay are $-10.1 \pm 2.6\%$ with 3.9σ

TABLE III: Branching ratios of $B \to \pi\pi, K\pi$and KK decays with $\phi_3 = 80^0$, $R_b = \sqrt{\rho^2 + \eta^2} = 0.38$. Here we adopted $m_0^\pi = 1.3$ GeV, $m_0^K = 1.7$ GeV and $0.36 < \omega_B < 0.44$. Unit is 10^{-6}.

Modes	CLEO	BELLE	BABAR	World Av.	PQCD
$\pi^+\pi^-$	$4.5^{+1.4+0.5}_{-1.2-0.4}$	$4.4 \pm 0.6 \pm 0.3$	$4.7 \pm 0.6 \pm 0.2$	4.55 ± 0.44	$5.93 - 10.99$
$\pi^+\pi^0$	$4.5^{+1.8+0.6}_{-1.6-0.7}$	$5.3 \pm 1.3 \pm 0.5$	$5.8 \pm 0.6 \pm 0.4$	5.20 ± 0.79	$2.72 - 4.79$
$\pi^0\pi^0$	< 4.4	$2.32^{+0.44+0.22}_{-0.48-0.18}$	$1.7 \pm 0.32 \pm 0.10$	2.01 ± 0.43	$0.1 - 0.65$
$K^\pm\pi^\mp$	$18.0^{+2.3+1.2}_{-2.1-0.9}$	$18.5 \pm 1.0 \pm 0.7$	$17.9 \pm 0.9 \pm 0.7$	18.2 ± 0.8	$12.67 - 19.30$
$K^0\pi^\mp$	$18.8^{+3.7+2.1}_{-3.3-1.8}$	$22.0 \pm 1.9 \pm 1.1$	$26.0 \pm 1.3 \pm 1.0$	22.3 ± 1.4	$14.43 - 26.26$
$K^\pm\pi^0$	$12.9^{+2.4+1.2}_{-2.2-1.1}$	$12.8 \pm 1.4^{+1.4}_{-1.0}$	$12.0 \pm 0.7 \pm 0.6$	12.6 ± 1.1	$7.87 - 14.21$
$K^0\pi^0$	$12.8^{+4.0+1.7}_{-3.3-1.4}$	$12.6 \pm 2.4 \pm 1.4$	$11.4 \pm 0.9 \pm 0.6$	12.3 ± 1.7	$4.46 - 8.06$
$K^\pm K^\mp$	< 0.8	< 0.7	< 0.6	< 0.6	0.06
$K^\pm \bar{K}^0$	< 3.3	< 3.4	$1.45 \pm 0.50 \pm 0.11$	$1.45 \pm 0.50 \pm 0.11$	1.4
$K^0 \bar{K}^0$	< 3.3	< 3.2	$1.19 \pm 0.38 \pm 0.13$	$1.19 \pm 0.38 \pm 0.13$	1.4

TABLE IV: Branching ratios of $B \to \phi K^{(*)}$and $K^*\pi$ decays with $\phi_3 = 80^0$, $R_b = \sqrt{\rho^2 + \eta^2} = 0.38$. Here we adopted $m_0^\pi = 1.3$ GeV and $m_0^K = 1.7$ GeV. Unit is 10^{-6}.

Modes	CLEO	BELLE	BABAR	World Av.	PQCD
$\phi K^\pm$	$5.5^{+2.1}_{-1.8} \pm 0.6$	$9.4 \pm 1.1 \pm 0.7$	$10.0^{+0.9}_{-0.8} \pm 0.5$	9.3 ± 0.8	$8.1 - 14.1$
ϕK^0	$5.4^{+3.7}_{-2.7} \pm 0.7$	$9.0 \pm 2.2 \pm 0.7$	$7.6^{+1.3}_{-1.2} \pm 0.5$	7.7 ± 1.1	$7.6 - 13.3$
$\phi K^{*\pm}$	$10.6^{+6.4+1.8}_{-4.9-1.6}$	$6.7^{2.1+0.7}_{-1.9-1.0}$	$12.1^{+2.1}_{1.9} \pm 1.1$	9.4 ± 1.6	$12.6 - 21.2$
ϕK^{*0}	$11.5^{+4.5+1.8}_{-3.7-1.7}$	$10.0^{+1.6+0.7}_{-1.5-0.8}$	$11.1^{+1.3}_{-1.2} \pm 0.8$	10.7 ± 1.1	$11.5 - 19.8$
$K^{*0}\pi^\pm$	$7.6^{+3.5}_{-3.0} \pm 1.6$	$19.4^{+4.2+4.1}_{-3.9-7.1}$	$15.5 \pm 3.4 \pm 1.8$	12.3 ± 2.6	$10.2 - 14.6$
$K^{*\pm}\pi^\mp$	$16^{+6}_{-5} \pm 2$	< 30	$-$	16 ± 6	$8.0 - 11.6$
$K^{*+}\pi^0$	< 31	$-$	$-$	< 31	$2.0 - 5.1$
$K^{*0}\pi^0$	< 3.6	< 7	$-$	< 3.6	$1.8 - 4.4$

deviations from zero in Belle Coll., and $-13.3 \pm 3.1\%$ with 4.2σ, which is in a good agreement with PQCD result[38].

VI. EXTRACTION OF $\phi_2(=\alpha)$ FROM $B \to \pi^+\pi^-$

Even though isospin analysis of $B \to \pi\pi$ can provide a clean way to determine ϕ_2, it might be difficult in practice because of the large uncertainty of the branching ratio of $B^0 \to \pi^0\pi^0$. In reality in order to determine ϕ_2, we can use the time-dependent rate of $B^0(t) \to \pi^+\pi^-$. Since penguin contributions are sizable about 20-30 % of the total amplitude, we expect that direct CP violation can be large if strong phases are different in the tree and penguin diagrams.

In our analysis we use the c-convention. The ratio between penguin and tree amplitudes is $R_c = |P_c/T_c|$ and the strong phase difference between penguin and tree amplitudes $\delta = \delta_P - \delta_T$. The time-dependent asymmetry measurement provides two equations for $C_{\pi\pi}$ and $S_{\pi\pi}$ in terms of three unknown variables R_c, δ and ϕ_2[49]. Since PQCD provides us $R_c = 0.23^{+0.07}_{-0.05}$ and $-41^o < \delta < -32^o$, the allowed range of ϕ_2 at present stage is determined as $55^o < \phi_2 < 100^o$ as shown in Figure VI.

According to the power counting rule in the PQCD approach, the factorizable annihilation contribution with large imaginary part becomes subdominant and give a negative strong phase from $-i\pi\delta(k_\perp^2 - x\,M_B^2)$. Therefore we have a relatively large strong phase in contrast to the QCD-factorization ($\delta \sim 0^o$) and predict large direct CP violation effect in $B^0 \to \pi^+\pi^-$ with $A_{cp}(B^0 \to \pi^+\pi^-) = (23 \pm 7)\%$, which will be tested by more precise experimental measurement

Quatity	Experiment	PQCD	QCDF[45]
$\frac{Br(\pi^+\pi^-)}{Br(\pi^\pm K^\mp)}$	0.25 ± 0.04	$0.30 - 0.69$	$0.5 - 1.9$
$\frac{Br(\pi^\pm K^\mp)}{2Br(\pi^0 K^0)}$	1.05 ± 0.27	$0.78 - 1.05$	$0.9 - 1.4$
$\frac{2\,Br(\pi^0 K^\pm)}{Br(\pi^\pm K^0)}$	1.25 ± 0.22	$0.77 - 1.60$	$0.9 - 1.3$
$\frac{\tau(B^+)}{\tau(B^0)}\,\frac{Br(\pi^\mp K^\pm)}{Br(\pi^\pm K^0)}$	1.07 ± 0.14	$0.70 - 1.45$	$0.6 - 1.0$

TABLE V: Ratios of CP-averaged rates in $B \to K\pi, \pi\pi$ decays with $\phi_3 = 80^0$, $R_b = 0.38$. Here we adopted $m_0^\pi = 1.3$ GeV and $m_0^K = 1.7$ GeV.

Direct $A_{CP}(\%)$	BELLE	BABAR	PQCD	QCDF
$\pi^+\pi^-$	$58 \pm 15 \pm 7$	$9 \pm 15 \pm 4$	$16.0 \sim 30.0$	-6 ± 12
$\pi^+\pi^0$	$-14 \pm 24^{+5}_{-4}$	$1 \pm 10 \pm 2$	0.0	0.0
$\pi^0\pi^0$	$43 \pm 51^{+17}_{-16}$	$12 \pm 56 \pm 6$	$20.0 \sim 40.0$	$-$
π^+K^-	$-10.1 \pm 2.5 \pm 0.5$	$-13.3 \pm 3.0 \pm 0.9$	$-12.9 \sim -21.9$	5 ± 9
π^0K^-	$4 \pm 5 \pm 2$	$6.0 \pm 6.0 \pm 1.0$	$-10.0 \sim -17.3$	7 ± 9
$\pi^-\bar{K}^0$	7^{+9+1}_{-8-3}	$-8.7 \pm 4.6 \pm 1.0$	$-0.6 \sim -1.5$	1 ± 1
π^0K^0	$16 \pm 29 \pm 5$	$-6 \pm 18 \pm 6$	$-0.90 \sim -1.03$	$-3.6 \sim 0.8$

TABLE VI: CP-asymmetry in $B \to K\pi, \pi\pi$ decays with $\phi_3 = 40^0 \sim 90^0$, $R_b = \sqrt{\rho^2 + \eta^2} = 0.38$. Here we adopted $m_0^\pi = 1.3$ GeV and $m_0^K = 1.7$ GeV.

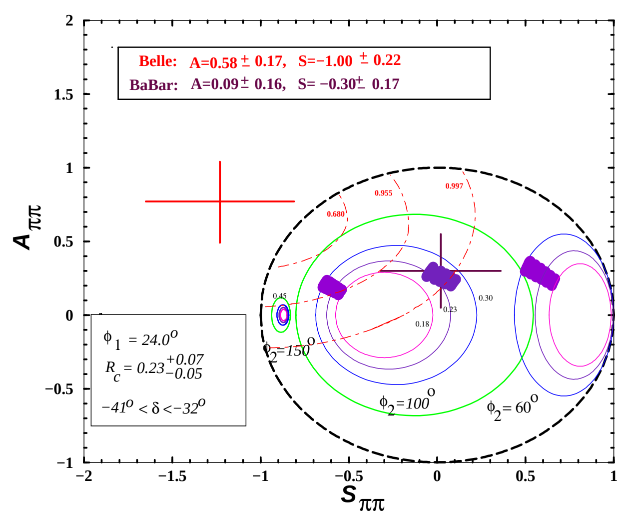

FIG. 10: Plot of $A_{\pi\pi}$ versus $S_{\pi\pi}$ for various values of ϕ_2 with $\phi_1 = 24.3^o$, $0.18 < R_c < 0.30$ and $-41^o < \delta < -32^o$ in the pQCD method.

within two years.

In the numerical analysis, though the data by Belle collaboration[54] is located ourside allowed physical regions, we considered the averaged value of recent measurements[54, 55]:

- $S_{\pi\pi} = -0.30 \pm 0.17 \pm 0.03$ (BaBar), $S_{\pi\pi} = -1.00 \pm 0.21 \pm 0.07$ (Belle);
- $A_{\pi\pi} = 0.09 \pm 0.15 \pm 0.04$ (BaBar), $A_{\pi\pi} = 0.58 \pm 0.15 \pm 0.07$ (Belle).

The central point of averaged data corresponds to $\phi_2 = 78^o$ in the PQCD method. Even if the data

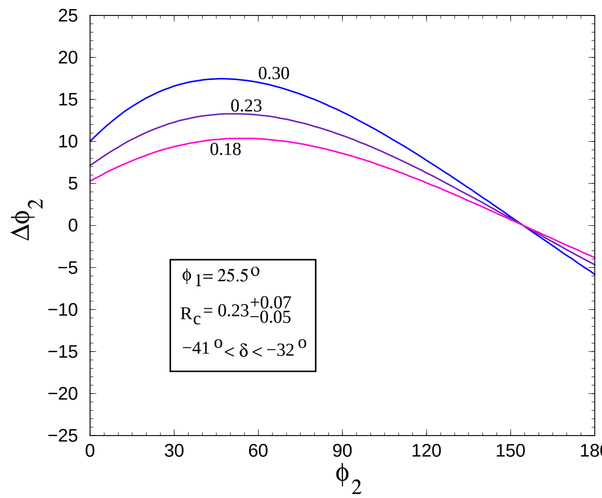

FIG. 11: Plot of $\Delta\phi_2$ versus ϕ_2 with $\phi_1 = 25.5^o$, $0.18 < R_c < 0.30$ and $-41^o < \delta < -32^o$ in the PQCD method.

by Belle collaboration[54] is located ourside allowed physical regions, we can have allowed ranges with 2 σ bounds, but large negative δ and $R_c > 0.4$ is prefered[52].

VII. EXTRACTION OF $\phi_3(=\gamma)$ FROM $B^0 \to K^+\pi^-$ AND $B^+ \to K^0\pi^+$

By using tree-penguin interference in $B^0 \to K^+\pi^-(\sim T' + P')$ versus $B^+ \to K^0\pi^+(\sim P')$, CP-averaged $B \to K\pi$ branching fraction may lead to non-trivial constaints on the ϕ_3 angle[56]. In order to determine ϕ_3, we need one more useful information on CP-violating rate differences[57]. Let's introduce the following observables :

$$\begin{aligned} R_K &= \frac{\overline{Br}(B^0 \to K^+\pi^-)\ \tau_+}{\overline{Br}(B^+ \to K^0\pi^+)\ \tau_0} = 1 - 2\ r_K\, cos\delta\ cos\phi_3 + r_K^2 \\ &\qquad\qquad\qquad\qquad\qquad \geq sin^2\phi_3 \end{aligned} \tag{14}$$

$$\begin{aligned} A_0 &= \frac{\Gamma(\bar{B}^0 \to K^-\pi^+ - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(B^- \to \bar{K}^0\pi^-) + \Gamma(B^+ \to \bar{K}^0\pi^+)} \\ &= A_{cp}(B^0 \to K^+\pi^-)\ R_K = -2r_K\, sin\phi_3\, sin\delta. \end{aligned} \tag{15}$$

where $r_K = |T'/P'|$ is the ratio of tree to penguin amplitudes and $\delta = \delta_{T'} - \delta_{P'}$ is the strong phase difference between tree and penguin amplitudes. After eliminate $sin\delta$ in Eq.(8)-(9), we have

$$R_K = 1 + r_K^2 \pm \sqrt{(}4r_K^2 cos^2\phi_3 - A_0^2 cot^2\phi_3). \tag{16}$$

Here we obtain $r_K = 0.201 \pm 0.037$ from the PQCD analysis[15, 59] and $A_0 = -0.11 \pm 0.065$ by combining recent measurements on CP asymmetry of $B^0 \to K^+\pi^-$: $A_{cp}(B^0 \to K^+\pi^-) = -11.7 \pm 2.8 \pm 0.7\%$ [54, 55] with present world averaged value of $R_K = 1.10 \pm 0.15$[58].

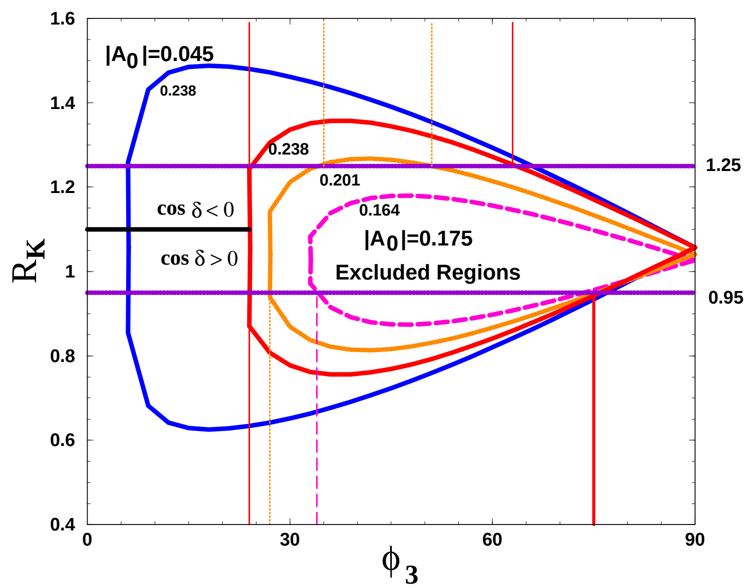

FIG. 12: Plot of R_K versus ϕ_3 with $r_K = 0.164, 0.201$ and 0.238.

As shown in Fig. 12, we can constrain ϕ_3 with $1\,\sigma$ range of World Averaged R_K as follows:

- For $cos\delta > 0$, $r_K = 0.164$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $24^o \leq \phi_3 \leq 75^0$.
- For $cos\delta > 0$, $r_K = 0.201$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $27^o \leq \phi_3 \leq 75^0$.
- For $cos\delta > 0$, $r_K = 0.238$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $34^o \leq \phi_3 \leq 75^0$.
- For $cos\delta < 0$, $r_K = 0.164$: we can exclude $0^o \leq \phi_3 \leq 6^0$.
- For $cos\delta < 0$, $r_K = 0.201$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $35^o \leq \phi_3 \leq 51^0$.
- For $cos\delta < 0$, $r_K = 0.238$: we can exclude $0^o \leq \phi_3 \leq 6^0$ and $24^o \leq \phi_3 \leq 62^0$.

According to the table 2, since we obtain $\delta_{P'} = 157^o$ and $\delta_{T'} = 1.4^o$, the value of $cos\delta$ becomes negative, -0.91. Therefore the maximum value of the constraint bound for the ϕ_3 is strongly depend on the uncertainty of $|V_{ub}|$. When we take the central value of $r_K = 0.201$, ϕ_3 is allowed within the ranges of $51^o \leq \phi_3 \leq 129^o$, which is consistent with the results by the model-independent CKM-fit in the (ρ, η) plane.

VIII. RADIATIVE B-DECAYS ($B \to (K^*/\rho/\omega)\gamma$):

Radiative B-meson decays can provide the most reliable window to understand the framework of the Standard Model(SM) and to look for New Physics beyond SM by using the rich sample of B-decays.

In contrast to the inclusive radiative B-decays, exclusive processes such as $B \to K^*\gamma$ are much easier to measure in the experiment with a good precision[60].

Decay Modes	CLEO	BaBar	Belle
$\mathcal{B}\nabla(B \to K^{*0}\gamma)$ (10^{-5})	$4.55 \pm 0.70 \pm 0.34$	$4.23 \pm 0.40 \pm 0.22$	$4.09 \pm 0.21 \pm 0.19$
$\mathcal{B}\nabla(B \to K^{*\pm}\gamma)(10^{-5})$	$3.76 \pm 0.86 \pm 0.28$	$3.83 \pm 0.62 \pm 0.22$	$4.40 \pm 0.33 \pm 0.24$
$\mathcal{B}\nabla(B \to \rho^0\gamma)$ (10^{-6})	< 17	< 1.2	< 2.6
$\mathcal{B}\nabla(B \to \rho^+\gamma)$ (10^{-6})	< 13	< 2.1	< 2.7
$\mathcal{B}\nabla(B \to \omega\gamma)$ (10^{-6})		< 1.0	< 4.4
$\mathcal{A}_{CP}(B \to K^{*0}\gamma)$ (%)	$8 \pm 13 \pm 3$	$-3.5 \pm 9.4 \pm 2.2$	$-6.1 \pm 5.9 \pm 1.8$
$\mathcal{A}_{CP}(B \to K^{*+}\gamma)$ (%)			$+5.3 \pm 8.3 \pm 1.6$

TABLE VII: Experimental measurements of the averaged branching ratios and CP-violating asymmetries of the exclusive $B \to V\gamma$ decays for $V = K^*, \rho$ and ω.

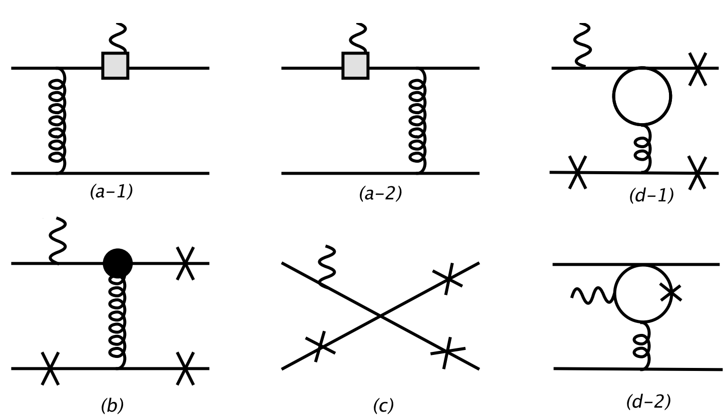

FIG. 13: Feynman diagrams of the magnetic penguin(a), chromomagnetic penguin(b), annihilation(c) and 0_2-penguin contributions for $B \to V\gamma$ decays

The main short-distance (SD) contribution to the $B \to K^*\gamma$ decay rate involves the matrix

element

$$< K^*\gamma|O_7|B >= \frac{em_b}{8\pi^2}(-2i)\epsilon_\gamma^\mu < K^*|\bar{s}\sigma_{\mu\nu}q^\nu(1-\gamma_5)b|B(p) >, \tag{17}$$

which is parameterized in terms of two invariant form fectors as

$$\begin{aligned} < K^*(P_3,\epsilon_3)|\bar{s}\sigma_{\mu\nu}q^\nu(1-\gamma_5)b|B(P) > &= [\epsilon_{3,\mu}(q\cdot P) - P_\mu(q\cdot\epsilon_3)]\cdot 2T_2(q^2) \\ &\quad + i\epsilon_{\mu\nu\alpha\beta}\epsilon_3^\nu P^\alpha q^\beta \cdot 2T_1(q^2). \end{aligned} \tag{18}$$

Here P and $P_3 = P - q$ are the B-meson and K^* meson momentum, respectively and ϵ_3 is the polarization vector of the K^* meson. For the real photon emission process the two form factors coincide, $T_1(0) = T_2(0) = T(0)$. This form factor can be calculable in the k_T factorization method including the sudakov suppression factor and the threshold resummation effects. As discussed in ref[63], we obtain $T(0) = 0.28\pm0.02$ for $B \to K^*\gamma$ which is far away from the QCD result 0.38 ± 0.06 by using the light-cone QCD sum rule [61], however in accordance with the preliminary result of Lattice QCD, 0.25 ± 0.06[62].

Even though theoretical predictions for the exclusive decays always has large model dependent hadronic uncertainties, such uncertainties can be cancelled in the searching of the CP-asymmetry and the isospin breaking effect.

Including all possible contributions from $0_{7\gamma}, 0_{8g}, 0_2$-penguin and annihilation in Figure. 13, we obtain the Branching ratios[63, 65]:

- $Br(B^0 \to K^{0*}\gamma) = (3.5^{+1.1}_{-0.8}) \times 10^{-5}$; $Br(B^+ \to K^{+*}\gamma) = (3.4^{+1.2}_{-0.9}) \times 10^{-5}$,
- $Br(B^0 \to \rho^0\gamma) = (0.95 \pm 0.14) \times 10^{-6}$; $Br(B^+ \to \rho^+\gamma) = (1.63 \pm 0.40) \times 10^{-6}$,

and the CP-Asymmetry :

- $Acp(B^0 \to K^{0*}\gamma) = (0.39^{+0.06}_{-0.07})\%$ $Acp(B^+ \to K^{+*}\gamma) = (0.62 \pm 0.13)\%$

The small difference in the branching fraction between $K^{0*}\gamma$ and $K^{+*}\gamma$ can be detected as the isopsin symmetry breaking which tells us the sign of the combination of the Wilson coefficients, C_6/c_7. We obtain

$$\Delta_{0-} = \frac{\eta_\tau Br(B \to \bar{K}^{0*}\gamma) - Br(B \to K^{*-}\gamma)}{\eta_\tau Br(B \to \bar{K}^{0*}\gamma) + Br(B \to K^{*-}\gamma)} = (5.7^{+1.1}_{-1.3} \pm 0.8)\% \tag{19}$$

where $\eta_\tau = \tau_{B^+}/\tau_{B^0}$. The first error term comes from the uncertainty of shape parameter of the B-meson wave function $(0.36 < \omega_B < 0.44)$ in charm penguin contribution and the second term

is origined from the uncertainty of η_T. By using the world averaged value of measurement and $\tau_{B^+}/\tau_{B^0} = 1.083 \pm 0.017$, we find numerically that $\Delta_{0-}(K^*\gamma)^{exp} = (3.9 \pm 4.8)\%$. In PQCD large isospin symmetry breaking in $B \to K^*\gamma$ system cannot be expected.

IX. SUMMARY AND OUTLOOK

In this paper I have summarized ingredients of k_T-factorization approach and some important theoretical predictions by comparing exparimental data, which is based on my previous works[13, 15, 16, 64]. The PQCD factorization approach provides a useful theoretical framework for a systematic analysis on non-leptonic two-body B-meson decays including radiative decays. Our results are in a good agreement with experimental data. Specially PQCD predicted large direct CP asymmetries in $B^0 \to \pi^+\pi^-, K^+\pi^-$ decays, which will be a crucial touch stone to distinguish our approach from others in future precise measurement. Recently the measurement of the direct CP asymmetry in $B \to K^\pm\pi^\mp$, $A_{cp}(K^+\pi^-) = -12 \pm 3\%$ is in accordance with our prediction.

We discussed the method to determine weak phases ϕ_2 within the PQCD approach through Time-dependent asymmetries in $B^0 \to \pi^+\pi^-$. We get interesting bounds on $55^o < \phi_2 < 100^o$ with 90% C.L. of the recent averaged measurements.

Acknowledgments It is a great pleasure to thank D.P. Roy, A. Kundu and Uma Shanka for their hospitality at WHEPP8-workshop, Mumbai in India. I wish to acknowlege the fruitful collaboration and joyful discussions with other members of PQCD working group. This work was supported in part by a visiting scholar program in DESY and in part by Grant-in Aid from NSC: NSC 92-2811-M-001-088 in Taiwan.

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